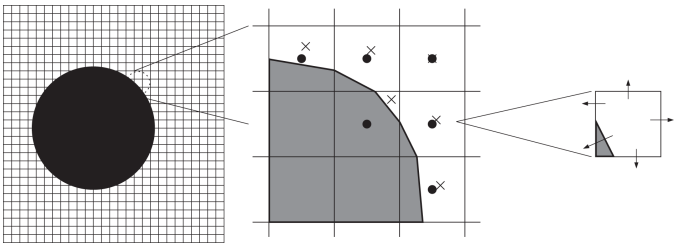
Master Project: Multigrid on Irregular Domains

Multigrid methods are the fastest known methods to solve elliptic boundary-value problems such as $\nabla^2\phi = 0$ (Laplace's equation). They can for example be used to compute electrostatic or gravitational potentials, to simulate incompressible flows, or to solve the heat equation. This project focuses on geometric multigrid methods, in which a hierarchy of structured grids is used. By applying an iterative method at different grid resolutions, the error is efficiently damped at both short and long wavelengths.

The goal of this project is to investigate and implement a multigrid method which can handle irregular boundaries, see the figure below. This would for example allow to include electrodes in an electrostatic solver or to simulate flow around an object. The project will consist of three parts:

1. Understand the basics of multigrid methods.
2. Explore literature on multigrid methods for irregular boundaries.
3. Implement one or more methods (based on simplicity, accuracy, performance) in an existing multigrid solver, and investigate their properties.



Example of an irregular domain, consisting of a rectangle minus a sphere. (Figure taken from Trebotich et al. 2014)

Why this project could be interesting:

- A number of papers has been written on the subject, but there is no clear best approach and there are plenty of open issues.
- Results could be applicable to many real-world problems.
- Gain hands-on experience with applied and numerical mathematics.
- Get familiar with multigrid methods, which are also widely used as preconditioners for other iterative matrix methods.

Contact person: Jannis Teunissen, jannis@teunissen.net, Multiscale Dynamics, CWI